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Cournot and Bertrand Game Models for A Simple Spectrum Sharing Framework in Cognitive Radio Networks

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Abstract— Cognitive radio technology has been proposed to achieve a more efficient spectrum usage by using spectrum opportunities in time, frequency and space which is not fully used by a licensed system (primary system), but without disturbing the primary system. In this paper, we address the problem of spectrum sharing among one primary user and two secondary users. We model this problem as a game and use Cournot and Bertrand game models for spectrum allocation to secondary users. In each game model we first present the formulation of static cases when the secondary users can observe the adopted strategies and the payoff of each other. However, this assumption may not be realistic in some cognitive radio systems. Therefore, we formulate dynamic approaches in which the secondary users just communicate with the primary user. The stability conditions of the dynamic behavior for these spectrum sharing schemes is investigated

Keywords- spectrum sharing; cognitive radio; game theory.

I. INTRODUCTION

Wireless spectral resources have become a very important attributes of society, thanks to the spread of new wireless communication technologies [1].

Recent measurements reveal that many portions of the licensed spectrum are not used during significant time periods [2]. Since the number of users and their data rates steadily increase, the traditional fixed spectrum policy is inefficient and is no longer a feasible approach. One proposal for alleviating the spectrum scarcity is allowing license-exempted Secondary Users (SU) to exploit the unused spectrum holes over some frequency ranges by using Cognitive Radio (CR) technology [3].

Dynamic spectrum sharing is a challenging problem in cognitive radio network due to the requirement of "peaceful" coexistence of both licensed and unlicensed users as well as the availability of wide range of radio spectrum.

In 2005 Haykin provided an introduction to cognitive radio techniques where the fundamental cognitive tasks were discussed [3]. Xing, Chandramouli and Cordeiro used a learning algorithm to obtain the Nash equilibrium in a competitive spectrum sharing model based on noncooperative game [4]. A demand responsive pricing framework was proposed to maximize profits of the legacy spectrum operators while, at the same time, regarding the users' response model to the operators' price strategy [5]. In 2007, the spectrum sharing was modeled using Cournot game in an Oligopoly market [6]. In the algorithm, players can simultaneously alter strategies after seeing the other player's move. As seen, it is shown that an iterative algorithm in the game can be used to converge to the unique equilibrium solution only in a few iterations.

In this paper, we consider the problem of dynamic spectrum sharing in a cognitive radio network. In such an environment, there is a primary user allocated with a licensed radio spectrum the utilization of which could be improved by sharing it with the secondary users. We formulate this spectrum sharing problem as an oligopoly market in which two secondary users compete with each other to gain the maximum profit. Cournot and Bertrand game models are used to analyze this situation and the Nash equilibrium is considered as the solution of these games. The main objective of these game formulations is to maximize the profit of all secondary users based on the equilibrium adopted by all secondary users.

We first consider the case where the secondary users can completely observe the strategies adopted by each other and the corresponding payoffs. Static games are used and the Nash equilibrium can be obtained in a centralized fashion. However, in some practical systems this assumption may not be valid (e.g., where the secondary users are out of transmission range of each other and they can only communicate with the primary user). For these scenarios, we use dynamic games in which selection of the strategy by a secondary user is based only on the information from the primary user. Then we study the stability conditions for the Nash equilibrium.

II. SYSTEM MODEL AND ASSUMPTIOS

A. *Primary and Secondary Users*

We consider a wireless system with a primary user and two secondary users who want to share the spectrum allocated to the primary user .In this case, the primary user is willing to share some portion of the spectrum (b_i) with secondary user i. The primary user charges the secondary user for the spectrum at a rate of c per unit bandwidth .The revenue of secondary user i is denoted by r_i per unit of achievable transmission rate (bit/sec).

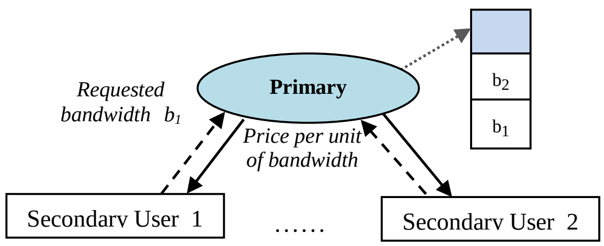

Figure 1. System model for spectrum sharing

B. *Spectral Efficiency*

The BER for an AWGN channel with MQAM modulation, ideal coherent phase detection, and SNR γ is bounded by [7]

$$\mathrm{BER} \le 0.2e^{-\frac{1.5\gamma}{M-1}} \quad (1)$$

Which is a tight bound good to within 1 dB for $M \ge 4$ and $0 \le \gamma \le 30$ dB.Where γ is the received SNR and M is the size of signal constellation. The spectral efficiency is defined as the number of bit/sec per unit bandwidth of the channel or equivalently $log_2 M$. Therefore, the spectral efficiency of the transmission for secondary user i can be obtained from (1):

$$k_i = log_2(1 + K\gamma_i) \quad (2)$$

Where

$$K = \frac{0.5}{ln\left(\frac{0.2}{BER}\right)} \quad (3)$$

γ_i is the SNR at the receiver of secondary user i,and BER is the bit-error-rate in Gaussian noise channel.We assume that the received SNR information is available at the transmitter by channel estimation.

III. SPECTRUM SHARING SCHEME

In this section, we formulate the problem of spectrum sharing among the primary user and secondary users as an oligopoly market competition. Static Cournot and Bertrand game models are presented for the ideal case where each secondary user can completely observe the strategies and the payoffs of the other secondary user. Afterwards, dynamic Cournot and Bertrand game models are presented for which the information of secondary users are unknown for each other.

A. *Static Cournot Game*

As in [6] a Cournot game can be formulated as follows. The players (i.e., firms in the oligopoly market) in this game are the secondary users. The strategy of each of the players corresponds to the allocated spectrum size (denoted by b_i for secondary user i) which is non-negative. The payoff for each player is the profit (i.e., revenue minus cost) of secondary user i (denoted by π^i) in sharing the spectrum with the primary user and other secondary users.

For the primary user, we assume that the pricing function used to charge the secondary users is given by

$$c = b_1 + b_2 \quad (4)$$

which is a uniform pricing scheme.

The revenue of the secondary user i can be obtained from $r_i \times k_i \times b_i$, while the cost of spectrum allocation is

$b_i \times c$. Therefore, the profit of the secondary users can be obtained as follows:

$$\begin{cases} \pi^1 = r_1 k_1 b_1 - c b_1 \\ \pi^2 = r_2 k_2 b_2 - c b_2 \end{cases} \quad (5)$$

In order to find the Nash Eq. point, we should solve these equations simultaneously

$$\begin{cases} \frac{\partial \pi^1}{\partial b_1} = 0 \\ \frac{\partial \pi^2}{\partial b_2} = 0 \end{cases} \quad (6)$$

By putting the cost from eq.(4) into eq.(5) and solving eq.(6) the optimal point is

$$\begin{pmatrix} b_1^* \\ b_2^* \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}^{-1} \begin{pmatrix} r_1 k_1 \\ r_2 k_2 \end{pmatrix} \quad (7)$$

As an example by setting the parameters as follows (which is held for other numerical results in this paper),

$$BER = 10^{-4}$$

$$\gamma_1 = 11\ dB$$

$$\gamma_2 = 10\ dB$$

$$r_1 = r_2 = 10$$

We get these optimal results

$$\begin{pmatrix} b_1^* \\ b_2^* \end{pmatrix} = \begin{pmatrix} 6.7657 \\ 4.4778 \end{pmatrix}$$

$$c^* = 11.2435$$

$$\begin{pmatrix} \pi^{1^*} \\ \pi^{2^*} \end{pmatrix} = \begin{pmatrix} 45.7744 \\ 20.0511 \end{pmatrix}$$

B. Dynamic Cournot Game

In a practical cognitive radio environment, secondary users may only be able to observe the pricing information from the primary user but not the strategies and profits of other secondary users. Therefore, we have to obtain Nash equilibrium of each secondary user only based on the interaction with the primary user. We investigate two dynamic methods, Bounded Rational method and Naive Expectation method.

1) Bounded Rational Method

Since all secondary users are rational to maximize their profits, they can adjust the spectrum size b_i based on the marginal profit function. In this case, each secondary user communicates with the primary user to obtain the differentiated pricing function for different strategies. The adjustment of the allocated spectrum size can be modeled as a repeated Cournot game as follows [6]:

$$b_i(t+1) = b_i(t) + \alpha_i b_i(t) \frac{\partial \pi^i(t)}{\partial b_i(t)} \qquad \text{□□□}$$

where $b_i(t+1)$ is the allocated spectrum size at time t+1, α_i is the speed adjustment parameter (i.e., learning rate) of secondary user i.

For simulation results we define an error , and do the iterations until the final point reaches the Nash Eq. point within the radius of this error. In other words, we do the iterations until the following condition is satisfied:

$$\frac{\|\underline{b}^* - \underline{b}(t)\|}{\|\underline{b}^*\|} < \varepsilon \qquad (9)$$

Where $\underline{b}^*$ is the Nash Eq. point and $\underline{b}(t)$ is the current strategy point in iteration t.

In Fig.2 the trajectory of iterations is sketched with parameters

$$\varepsilon = 0.001$$

$$\alpha_1 = \alpha_2 = 0.1$$

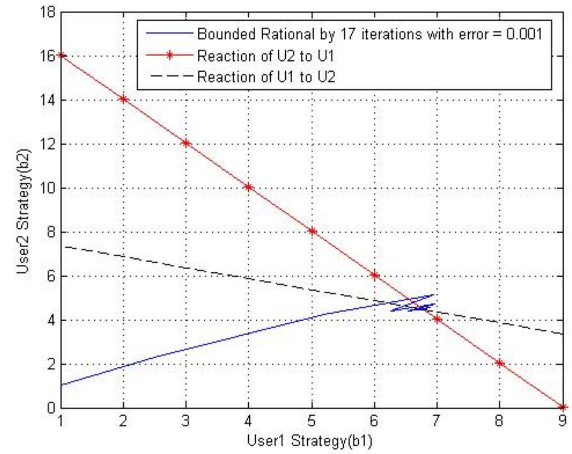

Figure 1. Trajectory of iterations in Bounded Rational Method

After 17 iterations the trajectory converged to the Nash point (within the neighborhood of radius ϵ).

2) Naive Expectation Method

In this case we assume that each secondary user assumes that the strategy of the other secondary user at time t+1 is the same as its strategy at time t ,i.e., player j assumes that:

$$b_i(t+1) = b_i(t) \qquad i \neq j \qquad (10)$$

Then by solving eq.(6) at each iteration we get a trajectory which is almost similar to the Bounded Rational Method. Such a figure is sketched in Fig.3. We see that for the same neighborhood radius the trajectory converges by 11 iterations (faster than Bounded Rational, but with greater fluctuations).

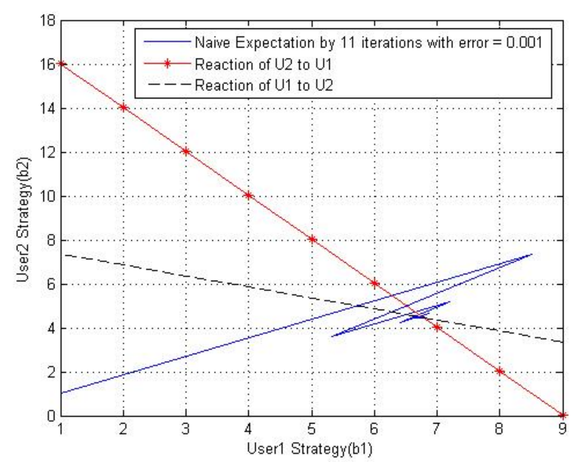

Figure 2. Trajectory of iterations in Naïve Expectation Method

3) Stability Analysis

Here, we consider the stability analysis of the Bounded Rational method by considering the eigenvalues of the Jacobian matrix of the mapping in eq.(8). By definition, the point of interest is stable if and only if the eigenvalues are all inside the unit circle of the complex plane (i.e., $|\lambda_i| < 1$ for i = 1,2) [8].

With two secondary users, there are two eigenvalues, and the Jacobian matrix can be expressed as follows

$$J(b_1, b_2) = \begin{bmatrix} \frac{\partial b_1(t+1)}{\partial b_1(t)} & \frac{\partial b_1(t+1)}{\partial b_2(t)} \\ \frac{\partial b_2(t+1)}{\partial b_1(t)} & \frac{\partial b_2(t+1)}{\partial b_2(t)} \end{bmatrix} =$$

$$\begin{bmatrix} 1+\alpha_1(r_1k_1 - 4b_1 - b_2) & -\alpha_1 b_1 \\ -\alpha_2 b_2 & 1+\alpha_2(r_2k_2 - 4b_2 - b_1) \end{bmatrix} \quad (11)$$

By solving

$$|eig(J(b_1^*, b_2^*))| < 1 \quad (12)$$

We find the appropriate region of α_1 and α_2 for stability which is sketched in Fig.4.

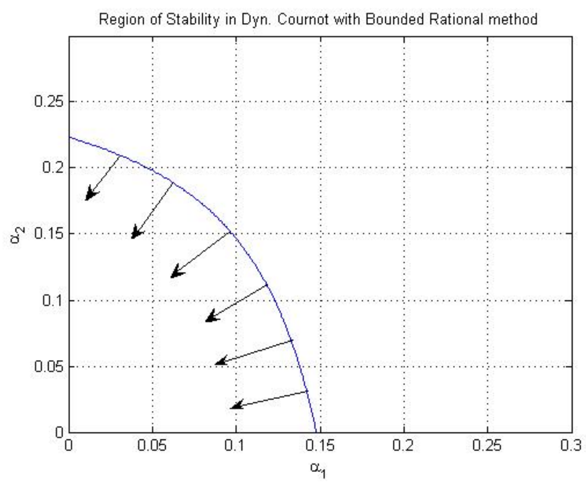

Figure 3. Region of values for α1 and α2 for stable Nash equilibrium

C. Static Bertrand Game

In the Bertrand game the strategy of each of the players corresponds to the price that they suggest to the primary user (denoted by p_i for secondary user i) which is non-negative. The revenue of the secondary user i is similar to the Cournot model, while the profit of the secondary users can be obtained as follows:

$$\begin{cases} \pi^1 = r_1k_1b_1 - p_1b_1 \\ \pi^2 = r_2k_2b_2 - p_2b_2 \end{cases} \quad (13)$$

We suppose that the quantity (amount of bandwidth) gained by each player is a linear function of its own price suggestion and the other player's price suggestion as follows

$$\begin{cases} b_1 = \overline{b_1} + \alpha_1 p_1 - \beta_1 p_2 \\ b_2 = \overline{b_2} + \alpha_2 p_2 - \beta_2 p_1 \end{cases} \quad (14)$$

It is reasonable that each player's quantity (b_i) is an increasing function of its price suggestion and a decreasing function of the other player's price suggestion.

Another assumption that we must take into consideration is that the sum of players' quantities is limited as follows

$$b_1 + b_2 \le S = 15 \quad (15)$$

here, we assumed that the upper bound for the sum of two players' bandwidth is 15 Mhz. In order to find the Nash Eq. point, we should solve these equations simultaneously

$$\begin{cases} \frac{\partial \pi^1}{\partial p_1} = 0 \\ \frac{\partial \pi^2}{\partial p_2} = 0 \end{cases} \quad (16)$$

By putting the quantities from eq.(14) into eq.(13) and solving eq.(16) the optimal point becomes

$$\begin{pmatrix} p_1^* \\ p_2^* \end{pmatrix} = \begin{pmatrix} 2\alpha_1 & -\beta_1 \\ -\beta_2 & 2\alpha_2 \end{pmatrix}^{-1} \begin{pmatrix} r_1k_1\alpha_1 - \overline{b_1} \\ r_2k_2\alpha_2 - \overline{b_2} \end{pmatrix} \quad (17)$$

As an example by setting the parameters as follows

$$\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = 0.5$$

$$\overline{b_1} = \overline{b_2} = 7$$

We get the following optimal results:

$$\begin{pmatrix} p_1^* \\ p_2^* \end{pmatrix} = \begin{pmatrix} 3.2466 \\ 2.4840 \end{pmatrix}$$

$$\begin{pmatrix} b_1^* \\ b_2^* \end{pmatrix} = \begin{pmatrix} 7.3813 \\ 6.6187 \end{pmatrix}$$

$$\begin{pmatrix} \pi^{1^*} \\ \pi^{2^*} \end{pmatrix} = \begin{pmatrix} 108.9673 \\ 87.6142 \end{pmatrix}$$

It is clearly seen from the above results that the player who suggests greater price, gets more quantity.

D. Dynamic Bertrand Game

Here, as in the dynamic Cournot game, we investigate two dynamic methods, Bounded Rational method and Naive Expectation method.

1) Bounded Rational Method

In this case, the adjustment of the price suggestion can be modeled as a repeated Bertrand game as follows:

$$p_i(t+1) = p_i(t) + s_i p_i(t) \frac{\partial \pi^i(t)}{\partial p_i(t)} \quad (18)$$

where $p_i(t+1)$ is the price suggestion of player i at time t+1, s_i is the speed adjustment parameter (i.e., learning rate) of secondary user i.

For simulation results we define an error , and do the iterations until the final point reaches the Nash Eq. point within the radius of this error. In other words, we do the iterations until the following condition is satisfied:

$$\frac{\left\|\underline{p}^{*}-\underline{p}(t)\right\|}{\left\|\underline{p}^{*}\right\|}<\ \varepsilon \qquad (19)$$

Where $\underline{p}^*$ is the Nash Eq. point and $\underline{p}(t)$ is the current point of iteration t. In Fig.5 the trajectory of iterations is sketched with parameters

$$\varepsilon = 0.001$$

$$s_1 = 0.4$$

$$s_2 = 0.5$$

After 10 iterations the trajectory converged to the Nash point(within the neighborhood of radius ϵ).

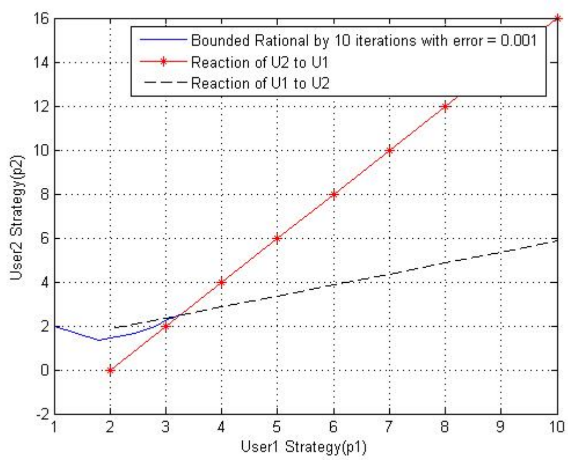

Figure 4. Trajectory of iterations in Bounded Rational Method

2) Naive Expectation Method

In this case we assume that each secondary user assumes that the strategies of other secondary users at time t+1 is the same as their strategies at time t ,i.e.,player j assumes that:

$$p_i(t+1) = \ p_i(t) \qquad i \neq j \qquad (20)$$

Then by solving eq.(16) at each iteration we get a trajectory sketched in Fig.6.We see that for the same neighborhood radius the trajectory converges by 11 iterations(slower than Bounded Rational, also with greater fluctuations).

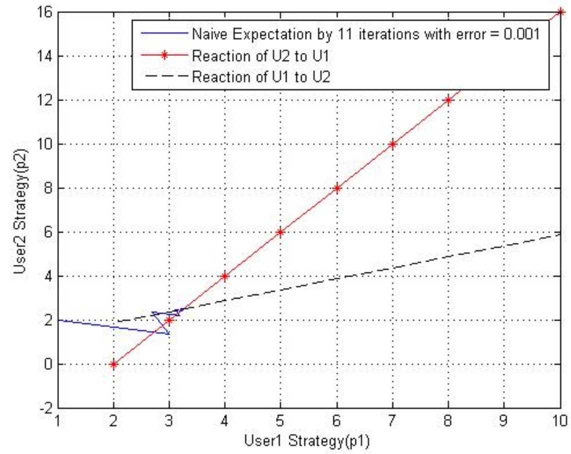

Figure 5. Trajectory of iterations in Naïve Expectation Method

3) Stability Analysis

Here, we consider the stability analysis of the Bounded Rational method by considering the eigenvalues of the Jacobian matrix of the mapping in eq.(18). With two secondary users, there are two eigenvalues, and the Jacobian matrix can be expressed as follows

$$J(p_1,p_2) = \begin{bmatrix} \frac{\partial p_1(t+1)}{\partial p_1(t)} & \frac{\partial p_1(t+1)}{\partial p_2(t)} \\ \frac{\partial p_2(t+1)}{\partial p_1(t)} & \frac{\partial p_2(t+1)}{\partial p_2(t)} \end{bmatrix} \qquad (21)$$

$$= \begin{bmatrix} 1+s_1(r_1k_1\alpha_1-\overline{b_1}-4\alpha_1p_1+\beta_1p_2) & s_1\beta_1p_1 \\ s_2\beta_2p_2 & 1+s_2(r_2k_2\alpha_2-\overline{b_2}-4\alpha_2p_2+\beta_2p_1) \end{bmatrix}$$

By solving

$$|eig(J(p_1^*,p_2^*))| < 1 \qquad (22)$$

We find the appropriate region of s_1 and s_2 for stability which is sketched in Fig.7.

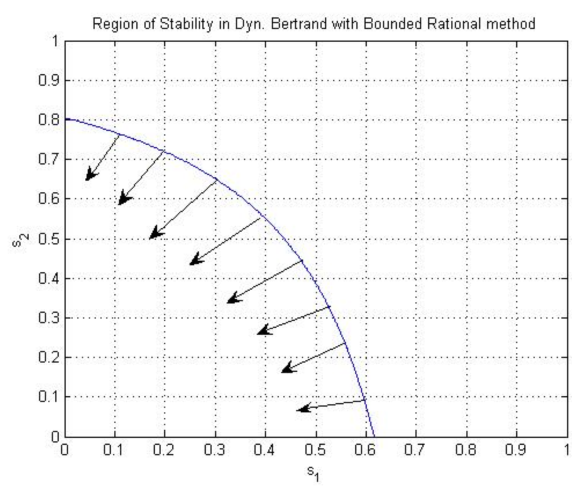

Figure 6. Region of values for s1 and s2 for stable Nash Eq.

IV. CONCLUSION

In this paper, we have investigated a simple competitive spectrum sharing scheme based on game theory for a cognitive radio network consisting of one primary user and two secondary users sharing the same frequency spectrum. We have modeled this spectrum sharing as an oligopoly market and static Cournot and Bertrand games have been used to analyze and obtain the Nash equilibrium However, these static games are useful only when each of the secondary users is able to observe the strategies and the payoffs of the other secondary user. Afterwards, we have presented dynamic games in which a secondary user adapts its spectrum sharing strategy by observing only the information offered by the primary user. We have analytically investigated the stability of these dynamic games using local stability theory. This spectrum sharing scheme will be useful for design and engineering of next generation cognitive wireless networks.

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