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Fingerprint Singular Point Detection via Quantization and Fingerprint Classification

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Abstract—This paper aims to present a fingerprint singular point detection algorithm and a rule-based fingerprint classification method. The singular point detection algorithm uses a quantization approach on the orientation field of the fingerprint image and seeks to locate the core and delta points via the changes of the gray levels around a 2x2 window. It has been found that with the application of an edge-trace-cum-core-delta-pairing algorithm and a merging-and-pruning heuristic as the post-processing steps, spurious singular points are removed and the final singular points are then used for classification. Fingerprint classification on NIST-4 database by rule-based method utilizes the number of singular points and three key geometry features to perform 5-class as well as 4-class classification using success rate (the accuracy) as the performance measure. It has been found to achieve 86.5% and 92.15% of success rate, respectively. The study has thus find the application of the new singular point detection algorithm via quantization and the rule-based classification to be promising as many of the fingerprint images in the NIST-4 database have been reported as poor quality, i.e. 22.35%.

Keywords-singular point; fingerprint singular point detection; fingerprint classification; quantization; fingerprint orientation field.

I. INTRODUCTION

Advancement in technology has allowed for more secured, efficient and reliable authentication such as by fingerprint recognition. Each individual has 10 fingerprints and there are thus millions of fingerprint templates. For the efficient search/retrieval of these templates, fingerprint images are generally sorted according to a few classes such as arch, tented arch, left-loop, right-loop and whorl as shown in Fig. 1. These classes can easily be identified by based on the global orientation field of the fingerprint. One such feature which can be extracted is called the singular point (SP). Fingerprint SP is considered as a landmark of fingerprint topology, it is scale, shift and rotation invariant [1]. An SP is a discontinuous point caused by the abrupt changes in the orientation field (OF) surrounding the point. Geometrically, a core is where the OF lines converge while a delta is where the lines show divergence. The detected core and delta points in a fingerprint image are then used to determine whether a fingerprint belongs to any of the five classes according to some specific rules, usually in terms of the numbers of SPs that have been detected and the orientation of the ridge lines around it.

All the classes have at least one core and one delta except the arch class which has none. Tented arch, left-loop, and right-loop each have a core and a delta, while generally a whorl has at least two cores (symbol ‘o’) and two deltas (symbol ‘Δ’) as shown in Fig. 1.

This paper first introduces how the SPs are detected and then assigned as either core or delta. The detection process is to be carried out before the classification of any fingerprint images. Various methods have been used for the detection of SPs. Reference [2] employs pixel-level singular point detection from multi-scale Gaussian filtered OF. Reference [3] uses topological structure for the analysis of singular points while [4] defines a new type of SPs consisting of both core and delta.

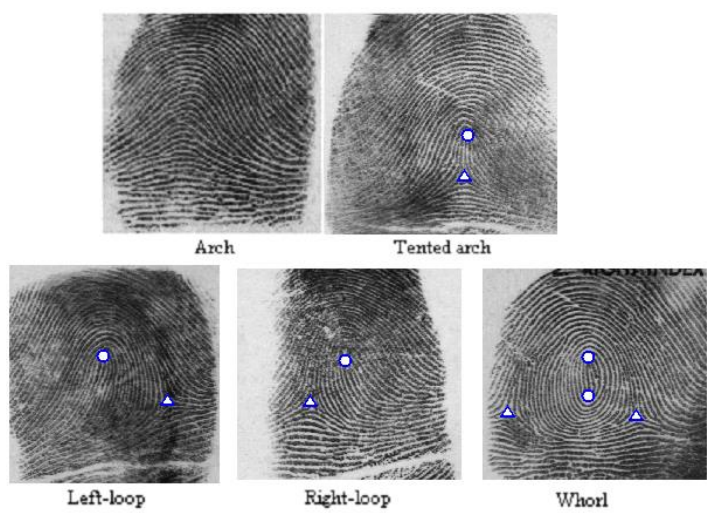


Figure 1. Fingerprint classes

The conventional method for SP detection as introduced by [5] is the Poincare index (PI) method. Researchers such as [6], [7], and [8] have employed this method in their work. The method describes the summation of the differences in OFs around the neighborhood of each pixel of interest (which is equal to the total rotation of a vector along a closed curve). A pixel is then identified as either a core or delta if the closed curve summation fulfils certain values. The PI method is highly dependent on the quality of the OF. Although it is efficient, the method is sensitive to noise [9].

Secondly, the paper introduces a rule-based classification method by employing the number of SPs and three key features based on the geometry around the SP region. A major challenge to fingerprint classification is the class variation. The majority of classification schemes use five classes. However, there is a wide variety of possible patterns within each class. Furthermore, in some cases, fingerprints from one class can appear very similar to fingerprints from another class. Therefore, there is large intra-class variation and small inter-class variation. This factor not only makes the classification of fingerprint hard but also a motivation for developing continuous classification scheme [10]. Fingerprint images taken from NIST special database 4 (NIST-4) have been used for evaluation of the classification's success rate in this paper [11]. Section II presents the fingerprint OF estimation by the well-known gradient-based method. Sections III and IV present the quantization approach while Sections V and VI the postprocessing step. Section VII presents the classification method and the experimental results are shown in Section VIII.

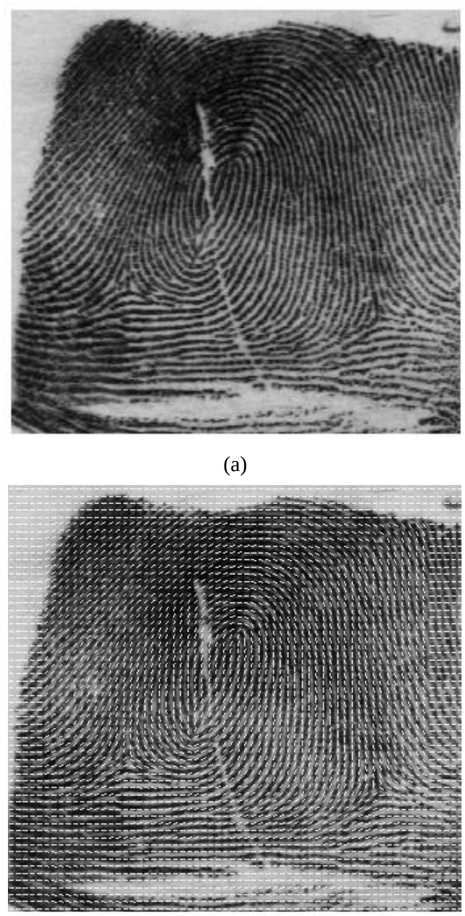
II. FINGERPRINT ORIENTATION ESTIMATION

Before fingerprint SP detection as well as classification can be carried out, the OF image has to be computed first. The common gradient-based method has been used. Although there exist a variety of methods to estimate OF of a fingerprint image, it was the gradient-based method as introduced in the work by [12], [13]; which has proven to be more accurate. References [4] and [2] have shown in their work that the gradient-based method obtained by squared averaging is an eigenvector of the auto-covariance matrix of gradients in the directions of x and y based on principal component analysis (PCA). The OF estimation is as follows:

1. The fingerprint image I is divided into square blocks.
2. Gradients in the x and y directions, g_x and g_y are computed for each pixel of I. The gradient vector is given by
$$g = g_x + jg_y \tag{1}$$
3. The local orientation θ of each block centered at each pixel is found using the square averaging over a local window of size w such that
$$\bar{g}^2 = \frac{1}{w^2}\left(\sum g_x^2 - g_y^2 + j\sum 2g_x g_y\right) \tag{2}$$
4. A median filtering of carried out to reduce noise effect on the square averaging and to remove blocky effects of the block operation and the local orientation is then computed:
$$\theta = \frac{1}{2}\tan^{-1}\left(\sum 2g_x g_y \Big/ \sum g_x^2 - g_y^2\right) \tag{3}$$
5. The local orientation is then smoothed by using a lowpass filter to obtain the smoothed OF image $\tilde{\theta}$. Note that before the smoothing is carried out, the orientation needs to be converted to a continuous vector field by doubling the angle (this is to avoid cancellation of the field having vectors opposite each other but aligned in the same direction. Fig. 2(a) shows an original fingerprint image and Fig. 2(b) the OF line superimposed on the fingerprint image and Fig. 2(c) is the smoothed OF image.

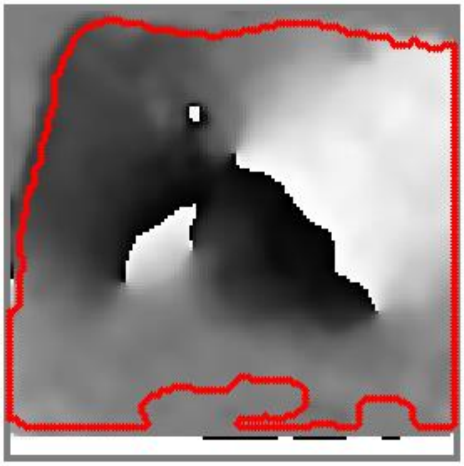
III. SINGULAR POINT DETECTION VIA QUANTIZATION

In the quantization method, the smoothed OF $\tilde{\theta}$ is quantized (digitized) into three gray levels as shown in Fig. 2. An SP is defined as a point where at least three gray levels (the pixel orientation) met as it clearly shows discontinuity. In the spatial coordinate for image, the SP is the point where in a 2x2 pixel region, there are at least three gray levels. This is as depicted in Fig. 2(d) and four such points have been labelled, SP(1) to SP(4).

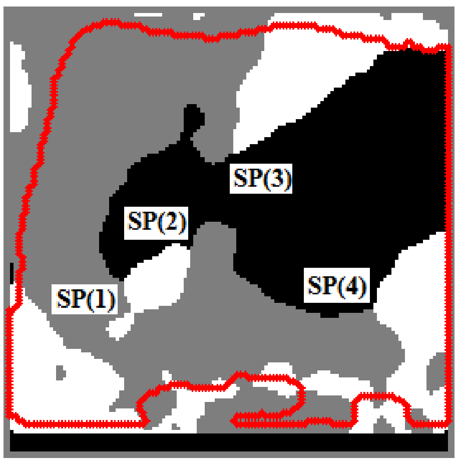


(a)

(b)



(c)



(d)

Figure 2. (a) fingerprint image, (b) orientation field (OF) superimposed on fimgerprint image (b) smoothed OF, (c) quantized OF

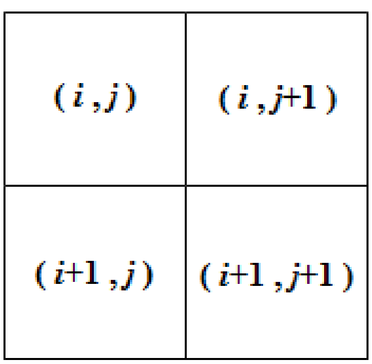


Figure 3. Region of the SP

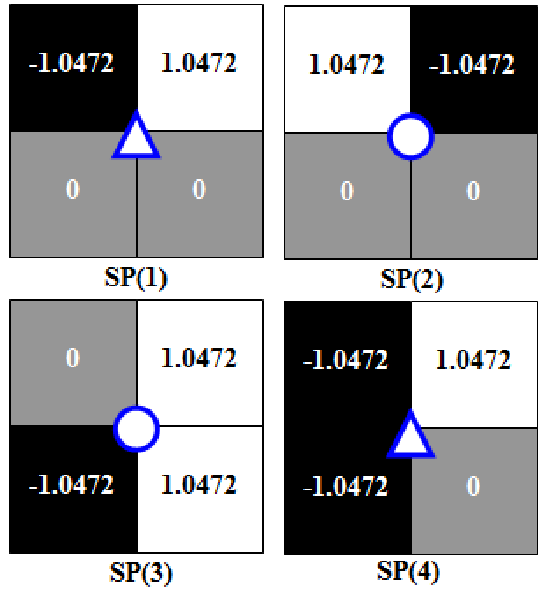


Figure 4. Closed up region of SP(1), SP(2), SP(3), SP(4)

To determine if an SP is a core or delta, the difference measure in the OF $\tilde{\theta}$ within the 2x2 pixel region of Fig. 3 following a clockwise direction has been defined as follows.

$$\begin{aligned} \tilde{\theta}_1' &= \theta(i, j+1) - \theta(i, j), \\ \tilde{\theta}_2' &= \theta(i+1, j+1) - \theta(i, j+1) \\ \tilde{\theta}_3' &= \theta(i+1, j) - \theta(i+1, j+1), \\ \tilde{\theta}_4' &= \theta(i, j) - \theta(i+1, j) \end{aligned} \quad (4)$$

If $\tilde{\theta}_k', k = 1,2,3,4$ is positive, then it is assigned a label of +1 (the label is denoted by L). If $\tilde{\theta}_k'$ is negative, then it is assigned a –1 to it. If no changes, assign zero. If the sum of all $\mathrm{L}\tilde{\theta}_k'$ is +1, then the SP is a core, if it is –1, then it is a delta.

$$SP = \text{Core iff} \sum_{k=1}^{4} \mathrm{L}\tilde{\theta}_k' = +1 \text{ or Delta iff} \sum_{k=1}^{4} \mathrm{L}\tilde{\theta}_k' = -1 \quad (5)$$

Based on Fig. 4, the calculations show that SP(1) and SP(4) are both deltas while SP(2) and SP(3) are both cores. The calculation for SP(1) is as shown below.

$$SP(1) = -1.0472 \to 1.0472 \to 0.000 \to 0.000 \to -1.0472$$

$$\mathrm{L}\tilde{\theta}_1' = +1 \;\; \mathrm{L}\tilde{\theta}_2' = -1 \, \mathrm{L}\tilde{\theta}_3' = 0 \;\; \mathrm{L}\tilde{\theta}_4' = -1$$

$$\sum_{k=1}^{4} \mathrm{L}\tilde{\theta}_k' = -1 \text{ is a delta}$$

IV. MULTI-LEVEL QUANTIZATION

The quantization approach can be extended to consider multiple level, i.e. above three grey levels. As a 2x2 region has only four pixels, the maximum gray levels are four if all pixels take different levels of intensity (the orientation values). A quantization of level above three may cause an SP to split into two, three, and so forth. The split of an SP may occur directly adjacent to each other (assume 8-connectedness), or a distance more than one pixel away. From observation and to make the approach simpler, the author assume a split if it occurs to be directly in the neighborhood. A simple connected component labeling will group all SPs of the same kind together. From the different groups of cores or deltas, only one core or one delta is picked. The picked core or delta is referred as the dominant core or dominant delta point. A dominant core or dominant delta must be the one with the largest OF difference as defined below.

Let $C=\{C1, C2, \dots, Cn\}$ and $D=\{D1, D2, \dots, Dm\}$ be a group of n cores and m deltas with same connected label. Then a dominant core $C(dom)$ or dominant delta $D(dom)$ is one of the core or delta satisfying the condition:

$$\begin{aligned}&\text{if } \tilde{\theta}'^{(Ck)}_{\max} = \max_{\text{all}\, C}\{\tilde{\theta}'_{\max}\}, \text{ then } C(dom) = Ck\\ &\text{if } \tilde{\theta}'^{(Dk)}_{\max} = \max_{\text{all}\, D}\{\tilde{\theta}'_{\max}\}, \text{ then } C(dom) = Dk\end{aligned} \qquad (6)$$

where $\tilde{\theta}'_{\max} = \max_{k=1,2,3,4}\{|\tilde{\theta}'_k|\}$, for $1 \le k \le n$ and $1 \le k \le m$ for core and delta, respectively. This approach for choosing a dominant point is referred to as the maximum-absolute-difference (MAD). Fig. 5 shows the regions of the SPs of Fig. 2 when level-5 quantization is applied. The final SPs chosen are the top points for both SP(1) and SP(2), the right point for SP(3) and the top-left point for SP(4) which can also be clearly shown by the contract of the quantized OF image.

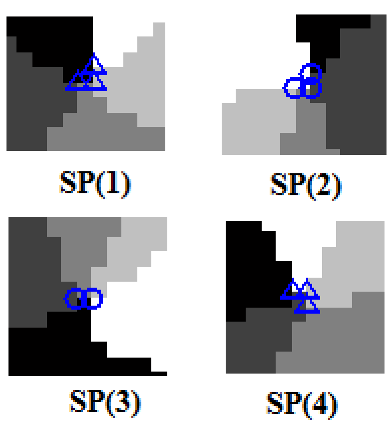


Figure 5. Closed up regions of SPs for level-5 quantization

V. EDGE-TRACE-CUM-CORE-DELTA-PAIRING

The dominant SPs detected by the SP detection algorithm via quantization may come with many false SPs mainly due to the quality of the fingerprint images or as the results of quantization as the level increases. Such singular points are also called spurious SPs and need to be removed in order to preserve good detection rate [14]. As such, an algorithm called the edge-trace-cum-core-delta pairing algorithm has been devised to eliminate the false SPs.

Firstly, all the points that fall within three pixels from the segmented border are rejected (segmentation is not discussed as falls outside the scope of this paper). This is to account for to the border and background transition effect which has caused many false SPs to arise. Secondly, a simple edge tracing process called edge-trace-cum-core-delta-pairing is carried out so that each SP is paired with another SP of different type. That is, a core must be paired with a delta. The pairing process starts from a single SP (either a core or a delta), and traces the edge line resulted from the quantization as illustrated in Fig. 6.

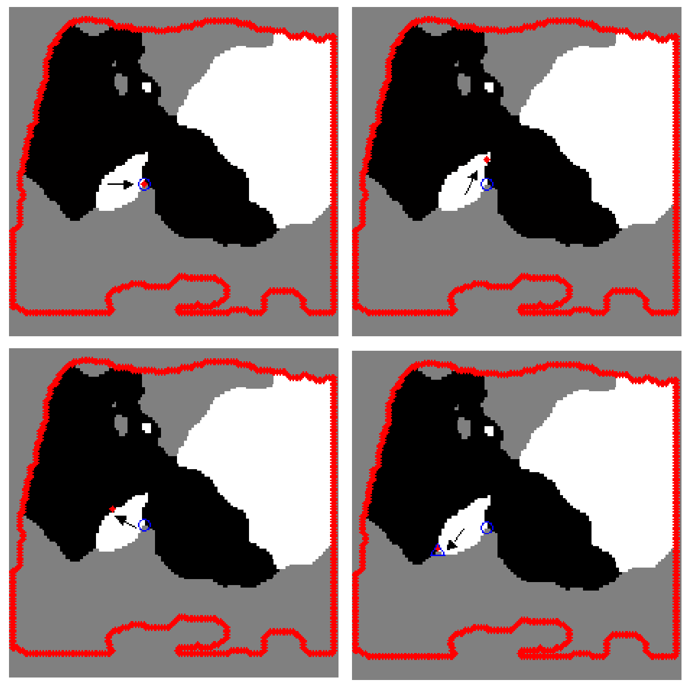


Figure 6. Edge tracing and core-delta pairing

The criterion of edge tracing is simply to find a MAD value from the four surrounding pixels of the dominant SP. Then by using the MAD value, the edge line is traced by moving either up, left, right and down along the same MAD computed from four pixels each time a movement is made. Once reaching the end point, it must either be a core or delta (e.g. if the starting point is a core, the SP point must be a delta). If the end point is of the same SP type which has already been detected by the SP detection algorithm, then do not pair both points, delete the current SP and move to the next SP for pairing.

VI. MERGING-AND-PRUNING HEURISTIC

Once the edge tracing is completed, the next step is to go through a merging-and-pruning heuristic. The heuristic is an elimination technique that has been devised by observing the ridge structure and regions around singularities of the quantized OF image of the fingerprints according to its various classes. The method is very practical, yet not guarantee to be optimal as there may still be some unobserved situations or the images are too noisy so as to render different results. The general processes of the heuristic are as follows:

1. Pruning process #1 – Prune all loops.
2. Merging process #1 – Merge short disjointed segment of the edge line within certain permitted range.
3. Pruning process #2 – Selection of dominant pairs: find three paired core-delta with maximum distances (i.e. the distance between the same core-delta pair), assign the first two pairs as dominant pairs and keep a three pair. Prune all other pairs.
4. Prune process #3 – Prune small segments: if a core-delta pair is a short segment with the delta some pixels above the core, then prune the point.
5. Merging process #3 – Border merge: find all the singular points that fall near the segmented border.
   i) Situation one – when the third core-delta pair does not exist.
   - if the two dominant pairs each has a core and a delta near the border, and both fall on the same side of the border, merge them into a single core-delta pair.
   - if either two cores and one delta or one core and two deltas of the two dominant pairs are detected near the border (one of the dominant pair edge line falls totally within the range set for the border), merge them into a single core-delta pair.

   ii) Situation two – when the third core-delta pair exists.
   - if the two dominant pairs each has a core and a delta near the border, and both fall on the same side of the border, merge them into a single core-delta pair. Elevate the third core-pair as the second dominant pair.
   - if the third core-delta pair and one of the dominant pairs each has a core and a delta near the border, and both fall on the same side of the border, merge them into a single core-delta pair.
   - if either two cores and one delta or one core and two deltas of the third core-delta pair and one of the dominant pairs are detected near the border (one of their edge line falls totally within the range set for the border), merge them into a single core-delta pair.
   - if two deltas of the dominant pairs are near the border as well a core of the third pair that is near the border, merge them if they fall at the same side of the border.
6. Pruning process #5 – Border prune: find all the core points that fall near the segmented border.
   i) if its corresponding delta is also within the border, prune it. If the pruned core-delta point is a dominant pair, upgrade the third pair as the second dominant pair.

   ii) if the core-delta is a short segment, prune it and upgrade the third pair as the second dominant pair if it exists. However, if the third pair is also detected within the border, prune the third pair instead.
7. Pruning process #6 – centroid prune: if core is more than a certain radius away from the centroid of the segmented image and its core-delta distance is short while a third core-delta pair exists and not anywhere near the border, prune the core-delta pair and upgrade the third pair as a dominant pair.
8. Pruning process #7 – The final pruning.
   i) Situation two – two pairs of core-delta.
   - if two dominant pairs are left and one of the edge line is directly above the other with the associated delta above the core, prune it.
   - if both pairs have deltas on the left of its cores or on the right and that one of the edge line is directly above the other, prune the pair with the shorter edge line.
   - in a bounding box created by the two cores, if both deltas are bounded, then delete the pair where it has a delta above its core.

   ii) Situation one – only a pair of core-delta.
   - if only a single dominant pair is left, and if the delta point is above the core point, prune it.

Fig. 7 shows the results of the postprocessing (the edge-trace-cum-core-delta-pairing algorithm and the merging-and-pruning heuristic) where the spurious SPs are eliminated.

VII. FINGERPRINT CLASSIFICATION

Fingerprint classification takes into account the number of SPs after the merging-and-pruning heuristic, either no SP, a pair of SP or two pairs of SPs.

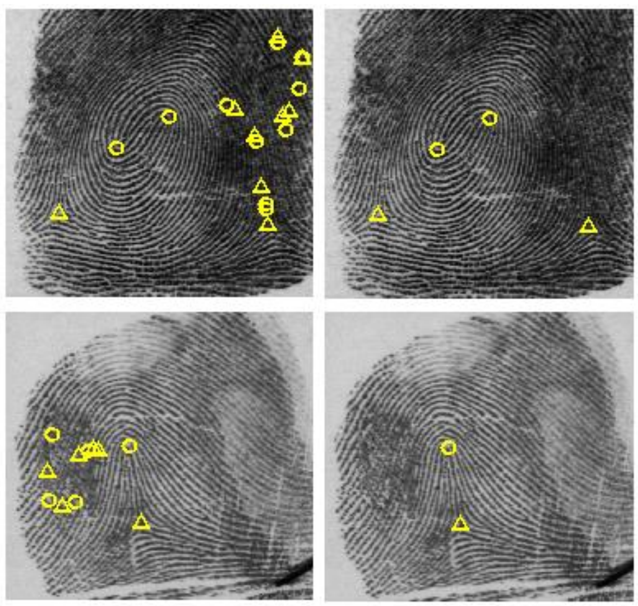


Figure 7. Results of the postprocessing

The classification algorithm also works on a simple rule-based method using three key features of the SPs. They are the orientation angle of core θ_C, orientation angle of delta θ_D and a length l as depicted in Fig. 8. For 5-class classification scheme:

1. If two dominant pairs exist, classify the fingerprint as a Whorl type.
2. If only a single dominant pair exists, do the following:
    i. if absolute angle θ_C is between 0° and 45°, calculate l.
    ii. if l is less than 15pixs or θ_C is less 22.5deg or θ_D is less 10deg, classify the fingerprint as a Tented arch type.
    iii. if angle θ_C is bigger than 22.5deg, classify the fingerprint as a Right Loop type.
    iv. if angle θ_C is smaller than -22.5deg, classify the fingerprint as a Left Loop type.
3. If no dominant pairs exist, classify the fingerprint as an Arch type.

All the measurements are chosen based on the image of size 128x128. All original images from NIST-4 have size of 512x512 but due to block size operation of size 4x4 for computing the OF, the final image is of size 128x128.

The performance measure uses the success classification rate or accuracy of classes for comparison.

$$\text{Success rate}, SR = \frac{\text{number of correctly assigned classes}}{\text{total number of images}}$$

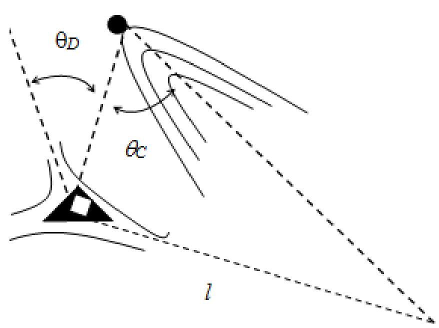


Figure 7. The three geometry features

TABLE I. SUCCESS RATE OF CLASSIFICATION

Quantization	Success Rate, SR	
	5-class	*4-class*
Level-3	85.65%	91.98%
Level-4	84.95%	91.6%
Level-5	**85.6%**	**92.15%**
Level-6	84.78%	91.7%
Level-7	84.65%	91.65%

TABLE II. 5-CLASS SUCCESS RATE OF CLASSIFICATION BY LEVEL-5 QUANTIZATION

True Class	Assigned Class				
	A	T	W	R	L
A	932	2	1	2	5
T	234	291	5	23	29
W	8	5	754	12	17
R	55	35	20	716	12
L	50	27	29	5	731

TABLE III. 4-CLASS SUCCESS RATE OF CLASSIFICATION BY LEVEL-5 QUANTIZATION

True Class	Assigned Class			
	A	W	R	L
A	1485	6	25	34
W	13	754	12	17
R	78	20	716	12
L	63	29	5	731

VIII. RESULTS AND DISCUSSION

Table I shows the tabulated classification's success rate according to different quantization levels, i.e. level-3 to level-7. It is based on NIST-4's 4000 images. The occurrence of the natural proportion of fingerprints according to class is said to be 31.7% for right loop, 33.8% for left loop, 27.9% for whorl, 3.7% for arch and 2.9% for tented arch [15]. Whorls and the loops group are the most common, making up 93.4%. Classification's success rate is based on the 5-class as well as the 4-class (as tented arches and arches constitute only a small proportion of the overall images, they are sometimes considered a single class). As some of the images from NIST-4 database are ambiguous due to difficulty in classification due to the condition of the fingerprints, some images have been assigned two classes. The assigned class is considered a match if it matches either one of the class.

The 5-class success rate is the highest for level-3 quantization while the 4-class for level-5 quantization. If a single quantization level is to be chosen, level-5 would be picked as the overall average between 5-class and 4-class is higher for level-5 quantization. Table II and Table III tabulate the results of level-5 quantization for 5-class and 4-class schemes, respectively.

The authors have also outlined some classification accuracy for the last 10 years as shown in Table IV. As observed, there are two common approaches to classification, i.e. the rule-based approach and the learning approach. In the learning approaches, only half of the images from the database will be classified as the first half will go through certain training process. As observed all the learning based approaches reported higher success rate for both 5-class and 4-class classification methods.

Surprisingly the rule-based method carried out by [24] has been reported to achieve 95.6% of success. Not only the work is a big accomplishment for a rule-based method but it also surpasses the learning based approaches without training. The method that was employed uses the singularity features as well as the global ridgelines of the fingerprint.

As the class proportion of NIST-4 is distributed evenly among all the five classes, the authors have also recalculated the success rate based on the natural proportion of the classes as reported by [15]. Upon recalculation, the 5-class has become 87.97% while the 4-class is 90.07%.

TABLE IV. OTHER RELATED WORK

Paper	Features	Classifier, Database	5-class	4-class
[16]	Fourier transform	NDA, 2H	90.7%	94%
[17]	Learned feature	Bayesian, 2H	91.6%	93.3%
[18]	Singularities and Gabor filters	SVM and naïve Bayes, 2H	90.8%	94.9%
[19]	Singularities and orientation field	SVM, 2H	93.5%	95%
[6]	Singularity feature	PDT, 2H	94.1%	95.7%
[20]	Singularity and ridges	Rule-based, W	84.3%	92.7%
[21]	Singularities and pseudoridges	Rule-based, W	84%	95.3%*
[22]	Orientation field flow	Rule-based, W	-	94.3%
[23]	Singularities and ridgelines	Rule-based, W	95.6%	-
[26]	Directional information of the thinned image	Rule-based, W	-	93.43%
This paper	Singularities and ridge orientation	Rule-based, W	85.65%	92.15%

Legends:

NDA: Nonlinear discrimination analysis
SVM: Support vector machine
PDT: Probabilistic decision tree
2H: uses images from the second half of NIST-4 database
W: uses all 4000 images from NIST-4 database
*: with reject rate of 11.8%

IX. CONCLUSIONS

In this paper a singular point (core or delta) detection algorithm via quantization of the orientation field image of the fingerprint and the selection of the dominant singular points using maximum-absolute-difference on the orientation field around a detected singular point is presented. It is followed by spurious singular points removal using edge-tracing-cum-core-delta-pairing algorithm and a merging-and-pruning heuristic. The classification algorithm works on a rule-based scheme based on the number of core-delta pairs. The number of singular points and three key geometry features have been used to classify tented arch, right loop, left loop, whorl and arch. The experimental results has shown that quantization at level-5 provides the best overall success rate. The 5-class success rate is 85.6% while the 4-class is above 92.15%. When success rate is recalculated to account for the natural distribution of the fingerprint classes [15], the 5-class rate is 87.97% while the 4-class is 90.07%. Hence, the application of the singular point detection algorithm via quantization with success classification rate above 85% shows promising as it has been reported that 22.35% of the NIST-4 images are of poor quality [25].

Reference [26] stated that no matter how definite fingerprint rules and pattern definitions are made, there will always be patterns concerning which there is doubt as to the classification they should be given. The primary reason for this is the fact that probably no two fingerprints will ever appear which are exactly alike. Other reasons are differences on the degree of judgment and interpretation of the individual classifying the fingerprints, the difference in the amount of pressure used by the person taking the prints, and the amount or kind of ink used. The classification of fingerprint impressions is thus a difficult work considering the variations which might exist even between two fingerprints of the same class (large intra-class variation) and similarities between different classes (small inter-class variation).

Automated classification system should be given its own set of formative framework without following the old classification rules which were first formed in the era where paper records are the norm. With the current advances in computing technology and intelligent techniques, it may not be that difficult to reclassify fingerprints.

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